Chapter 14 - Consumer's Surplus

→ The goal here will be to understand concepts that allow us to think about how well off a consumer is after a transaction

→ It's easiest to see this when goods are consumed in discrete amounts, so lets backtrack to chap 6 and think about demand for discrete goods.

Demand for a discrete good

→ consider the case where a consumer has a utility function defined over two goods, x_1 and x_2

→ let x_1 be a discrete good

→ that means this good can only be consumed in amounts that are integers.

→ let x_2 be a continuous good and the numeraire (so $P_2 = 1$)

→ Utility is thus given as:

$$u(x_1, x_2) = v(x_1) + x_2$$

→ and the budget constraint as:

$$P_1 x_1 + x_2 \leq m$$

→ Because x_1 is a discrete good, we don't have a marginal utility for x_1

→ you can't take the derivative w.r.t. x_1 since x_1 isn't continuous

→ What we use instead to determine demand are thresholds

→ e.g. we'll compare utility of consuming 0 units and 1 unit to see how much the consumer is willing to pay to consume 1 unit of x_1

→ the willingness to pay is the consumer's demand

→ we call the price at which the consumer is just indifferent between consuming another unit her <u>reservation price</u>

→ we solve for the reservation price of consuming one good, call it r_1, as:

$$\underbrace{v(0)+m}_{\text{utility from 0 } x_1 \text{ and all income spent on } x_2 \text{ (which has a price of 1)}} = \underbrace{m}_{\text{here we let } v(0)=0 \text{ as a normalization of utility}} = \underbrace{v(1)+m-r_1}_{\text{utility from consuming 1 unit of } x_1 \text{ and } m-r_1 \text{ units of } x_2}$$

→ solving this for r_1, we find

$$r_1 = v(1)$$

To find the reservation price on consuming 2 units of x_1:

$$v(1) + m - \underbrace{r_2}_{\text{one unit at price } r_2} = v(2) + m - \underbrace{2r_2}_{\text{2 units at price } r_2}$$

→ solve for r_2:

$$\Rightarrow r_2 = v(2) - v(1)$$

and you can do this for and number.

→ The ~~d~~ reservation prices; $r_1, r_2, r_3, \ldots$ ~~these~~ define the demand curve.

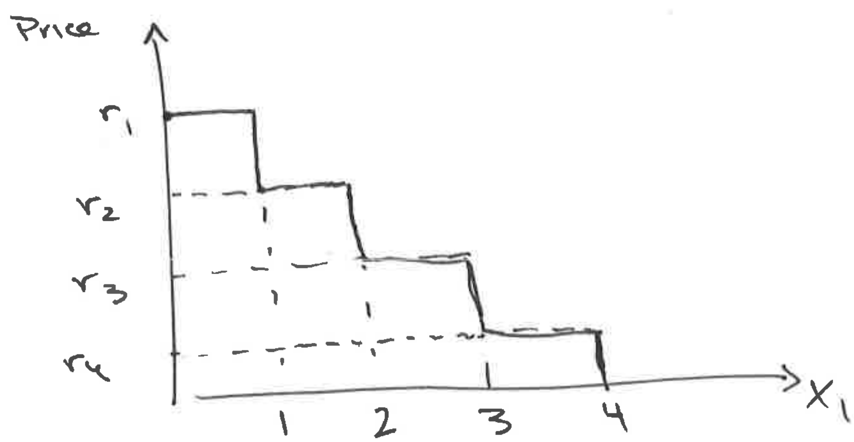
→ if ~~price~~ $p_1 > r_1$, demand $= 0$

if $r_2 < p_1 \leq r_1$, demand $= 1$

if $r_3 < p_1 \leq r_2$, demand $= 2$

and so on...

Graphically:



Constructing Utility From Demand

→ recall the reservation prices are defined as the differences in utility:

$$r_1 = v(1) - v(0)$$
$$r_2 = v(2) - v(1)$$
$$r_3 = v(3) - v(2)$$
$$\vdots$$

→ summing both sides of the equations we have:

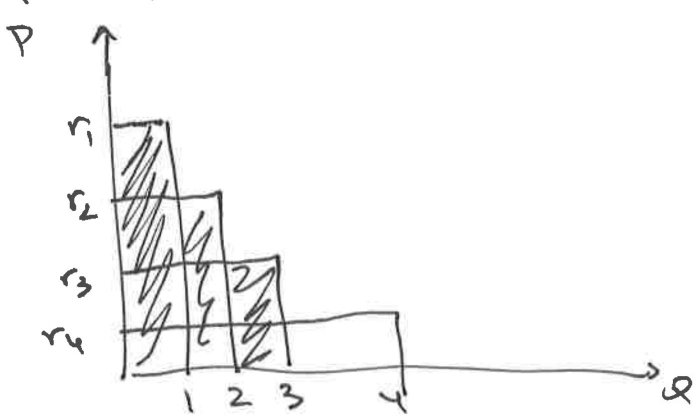
$$r_1 + r_2 + r_3 = \cancel{v(1)} - v(0) + \cancel{v(2)} - \cancel{v(1)} + v(3) - \cancel{v(2)} \cdots$$

$$r_1 + r_2 + r_3 = v(3) - \underbrace{v(0)}_{=0}$$

$$\Rightarrow v(3) = r_1 + r_2 + r_3$$

→ Generally: $v(n) = \sum_{i=1}^{n} r_i$

Graphically:



→ shaded area $= r_1 + r_2 + r_3$

→ this is called the <u>gross consumer's surplus</u>

→ note it is a gross not net amount

→ the net amount takes into account that by purchasing more of good one the consumer can't afford as much of good 2.

→ if the price of good 1 is given by p and the price of good 2 is 1 we have:

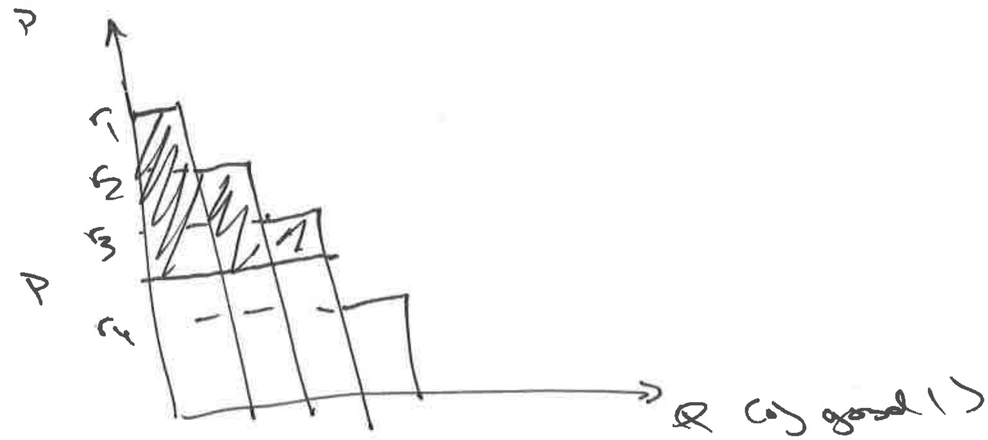
total utility = $v(n) + m - pn$

thus the net consumer's surplus (or just consumer's surplus) from consuming n units of good 1 is:

$$\underbrace{v(n) - pn}$$

measures utility from consuming n units of good 1 minus reduct. in expend on good 2

Graphically:



→ another way to think about this:

→ value unit n at r_n but pay just p for it.

→ Thus can find CS as:

$$CS = \underbrace{(r_1 - p)}_{} + \underbrace{(r_2 - p)}_{} + \underbrace{(r_3 - p)}_{} + \dots + (r_n - p)$$

net surplus from each unit consumed

$$CS = r_1 + r_2 + \dots + r_n + np$$

$$= \underbrace{v(n) - np}_{\text{same as before}}$$

→ and a 3rd way to think about this:

→ how much would the consumer need to be compensated to give up her entire consumption of the discrete good?

→ let this amount be R.

→ R would have to solve:

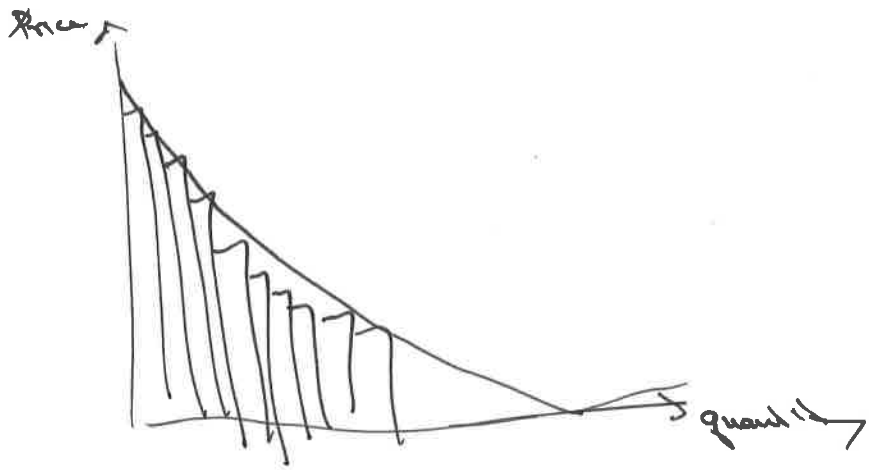
$$\underbrace{v(0)}_{=0 \text{ by dfn.}} + m + R = v(n) + m - pn$$

$$\Rightarrow R = v(n) - pn$$

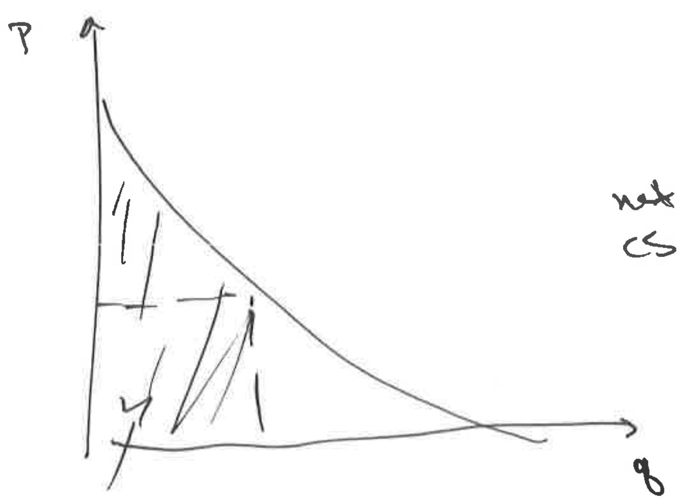
⇒ R is the consumer surplus

Continuous Demand an CS

→ Think about a continuous demand function ~~the~~ as an approximation of the discrete demand function:

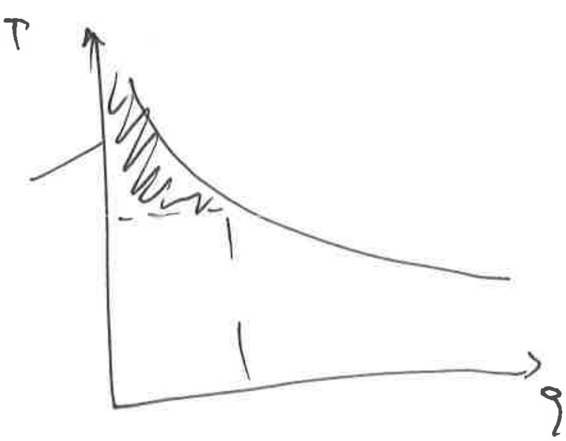


→ Then we can get ~~an approximate~~ values for the gross consumer's surplus and the net consumer's surplus in the same way:



Gross CS

net CS



A note of utility functions and consumer's surplus

→ Thus far, we've done all our consumer surplus w/ quasi-linear utility

→ quasi-linear utility is a special case

→ here, the individual's utility from consuming good one is independent of her cons. of good 2

→ Thus, reservation prices don't depend on cons. of good 2

→ Thus there is no income effect for good one

→ changing income doesn't change demand

→ B/c there are no income effects on good one w/ quasilinear utility we can measure CS for good one w/o considering demand for good 2

→ But, so long as income effects are small, we can use the same approach to get an approximate measure of CS from other utility functions that is pretty close to what the true CS is

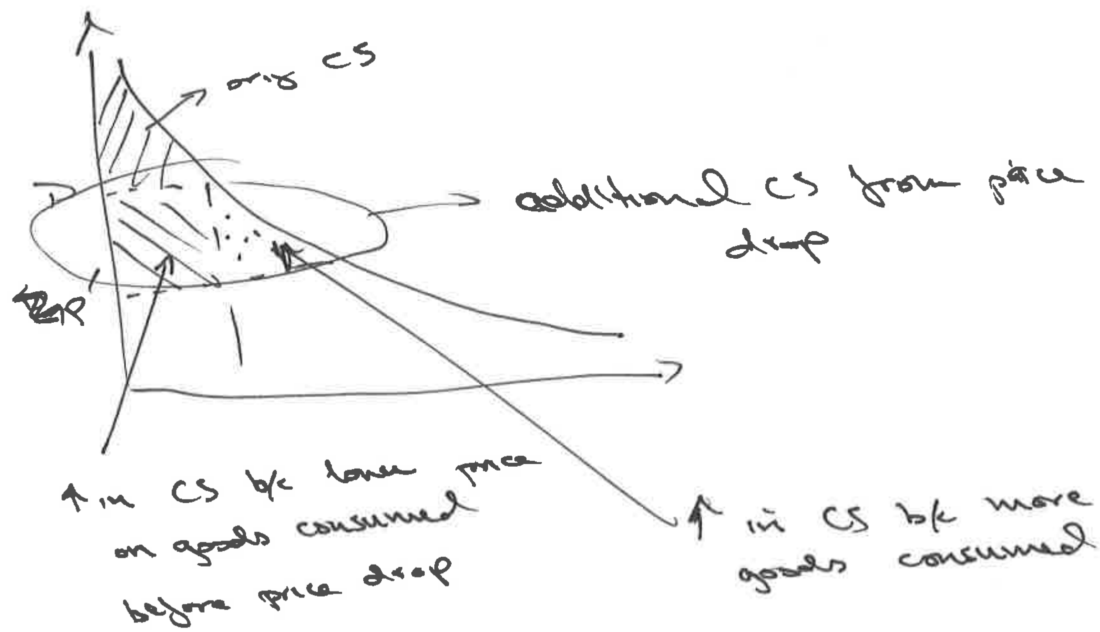
Changes in consumer surplus

→ Changes in CS come from 2 places:

1) paying a higher or lower price for the same quantity consumed

2) changes in the quantity consumed

→ consider price fall from p to p':



→ Numerical example:

→ let demand be given by $\underbrace{D(p) = 100 - 5p}_{\text{linear demand}}$

⇒ CS is triangular area

→ price moves from \$10 to \$5

→ To calc Δ in CS:

1) Demand before price Δ:

$100 - 5(10) = 100 - 50$
$= 50$

2) CS before price Δ:

→ area of triangle $= \frac{1}{2}bh = \frac{1}{2}(50)(\cancel{100}\,20 - 10)$ (20 = vertical intercept)

$= \frac{1}{2}(\cancel{5000}\ 50 \times 10) = \cancel{2500}$
$= \$250$

3) Demand after price Δ:

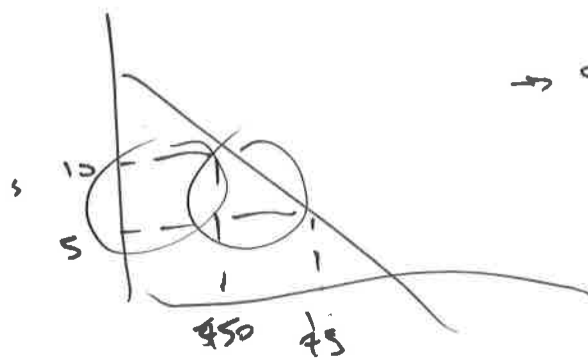
$D(p) = 100 - \cancel{5}(5)$
$= 100 - 25$
$= 75$

4) CS after change:

$\frac{1}{2}bh = \frac{1}{2}(75)(\overset{20}{\cancel{100}} - 5)$
$= \frac{1}{2}(75)(15)$
$= \$562.5$

5) Diff in CS = \$562.5 - \$250
= \$312.5

Alternatively, find area of trapezoid:



→ area of trapezoid = area ~~square~~ rectangle + area triangle

$= bh + \frac{1}{2}b_2h_2$
$= (5 \times 50) + (\frac{1}{2} \times 25 \times 5)$
$= 250 + 62.5$
$= 312.5$

Alternatives to CS for measuring changes in utility

→ Here we introduce the concepts of Compensating Variation and Equivalent Variation

→ There are 2 ways to measure a change in individual's utility that results from a price change

→ these put a dollar amount on the income the consumer would need to receive to be as well off after a price change

→ this is helpful to know when doing benefit-cost analysis, which is important in public policy, law, etc

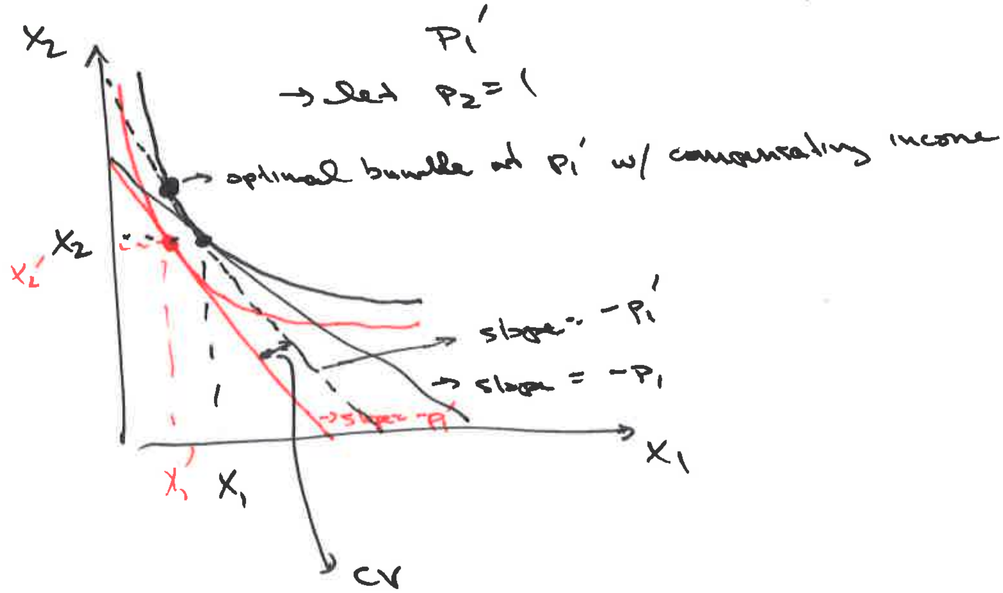
→ Compensating Variation is the change in income necessary to put the consumer on the same indifference curve as she was on before the price change.

→ i.e. income necessary to make consumer as well off after price change as before the price change.

Compensating variation (CV)

→ consider price increase from p_1 to p_1'

→ let $p_2 = 1$



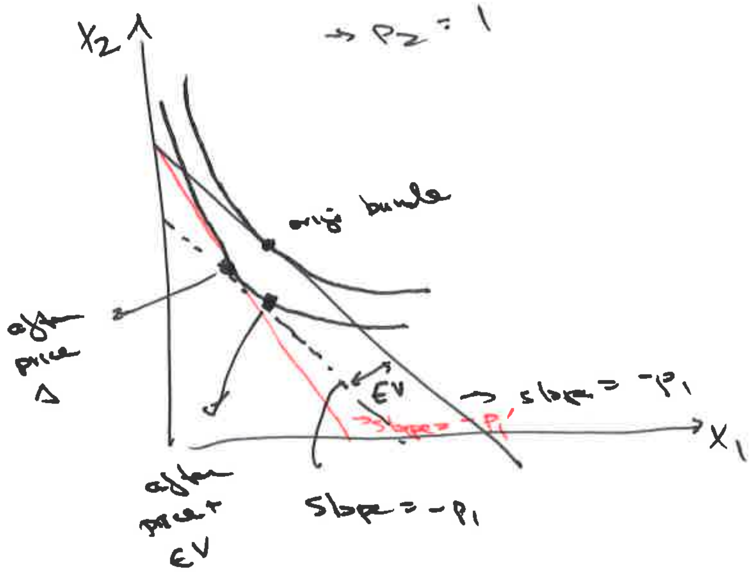
→ Equivalent variation is the amount of ~~income~~ money that would have to be taken away from the consumer before the price change to leave her as well off as she is after the price change

→ i.e. the change in income ~~necessary~~, ~~to keep~~ that is equivalent to the price change in terms of the effect on the consumer's utility.

Equivalent Variation (EV)

→ price p_1 ↑ to p_1'

→ $p_2 = 1$



→ EV ≠ CV in general

→ B/c consumption of 2 goods depends on price ratios and CV + EV compare indiff curves at diff prices

→ But w/ quasilinear the 2 are the same

→ in fact, w/ quasilinear CV = EV = CS

→ example:

let $u(x_1, x_2) = 4\ln(x_1) + x_2$

$p_1 = 2$, $p_2 = 1$

$m = \$80$

solve for demands:

→ if interior

$\frac{MU_1}{MU_2} = PR$

$\frac{1}{2}x_1^{-1/2} = 8$

$x_1^{-1/2} = 4$

$x_1 = 4^{-2}$

$x_1 = \frac{1}{4^2} = \frac{1}{16}$

⇒ $x_2 = m - (\frac{1}{16}x_2) = m - \frac{1}{8}$

$= \$80 - \frac{1}{8} = 79\frac{7}{8}$

To show that CV = EV = CS w/ quasilinear:

Note CV is the amount that solves

$$v(x_1') + \cancel{m} + CV - p_1'x_1' = v(x_1) + \cancel{m} - p_1 x_1$$

where x_1 is the optimal demand under price p_1 and x_1' the optimal demand under p_1'

$$\Rightarrow CV = v(x_1) - v(x_1') + p_1'x_1' - p_1 x_1$$

And EV solves:

$$v(x_1) + \cancel{m} - p_1 x_1 - EV = v(x_1') + \cancel{m} - p_1' x_1'$$

$$\Rightarrow EV = v(x_1) - v(x_1') + p_1'x_1' - p_1 x_1$$

and the change in CS can be found as

$$CS \text{ under } p_1 = v(x_1) - p_1 x_1$$

$$CS \text{ under } p_1' = v(x_1') - p_1' x_1'$$

$$\Rightarrow \Delta CS = CS(p_1) - CS(p_1')$$

$$= v(x_1) - p_1 x_1 - (v(x_1') - p_1' x_1')$$

$$= v(x_1) - v(x_1') + p_1' x_1' - p_1 x_1$$

→ all 3 measures are identical!

→ why? No income effects

Numerical Example

→ consider Cobb Douglas:

$$u(x_1, x_2) = x_1^{1/4} x_2^{3/4}$$

$$m = 10$$
$$p_1 = p_2 = 1$$

Demands: $x_1 = \frac{1}{4}\frac{m}{p_1}$

$$x_2 = \frac{3}{4}\frac{m}{p_2}$$

$$\Rightarrow x_1^* = \frac{1}{4}\frac{10}{1} = \frac{10}{4} = 2.5$$

$$x_2^* = \frac{3}{4}\frac{10}{1} = \frac{3 \times 10}{4} = \frac{30}{4} = 7.5$$

→ consider $p_1 \uparrow$ to $p_1 = 2$

To find CV, set utility after price change to that before and solve for m to afford this bundle:

utility before price Δ: $u(x_1^*, x_2^*) = 2.5^{1/4}\, 7.5^{3/4}$
$= 5.699$

Solve

$$\left(\frac{1}{4}\frac{m'}{2}\right)^{1/4}\left(\frac{3}{4}\frac{m'}{1}\right)^{3/4} = 5.699$$

$$m'\left(\frac{1}{8}\right)^{1/4}\left(\frac{3}{4}\right)^{3/4} = 5.699$$

$$m'(0.4792) = 5.699$$

$$m' = \frac{5.699}{0.4792} = 11.892$$

$\Rightarrow CV = m' - m = 11.892 - 10$

$= \boxed{1.892}$

$\rightarrow$ To find EV, find m' at prices $(1,1)$ that make as well off as m at prices $(2,1)$:

utility at prices $(2,1)$ w/ income m:

$$u(x_1, x_2) = \left(\frac{1}{4}\frac{m}{p_1}\right)^{1/4}\left(\frac{3}{4}\frac{m}{p_2}\right)^{3/4}$$

$$= \left(\frac{1}{4}\frac{10}{2}\right)^{1/4}\left(\frac{3}{4}\frac{10}{1}\right)^{3/4}$$

$$= \left(\frac{10}{8}\right)^{1/4}(7.5)^{3/4}$$

$$= 4.792$$

solve for m' from:

$$\left(\frac{1}{4}\frac{m'}{1}\right)^{1/4}\left(\frac{3}{4}\frac{m'}{1}\right)^{3/4} = 4.792$$

$$m'\left(\frac{1}{4}\right)^{1/4}\left(\frac{3}{4}\right)^{3/4} = 4.792$$

$$m'(\cancel{5.6}\ 0.5698) = 4.792$$

$$m' = \frac{4.792}{0.5698} = 8.409$$

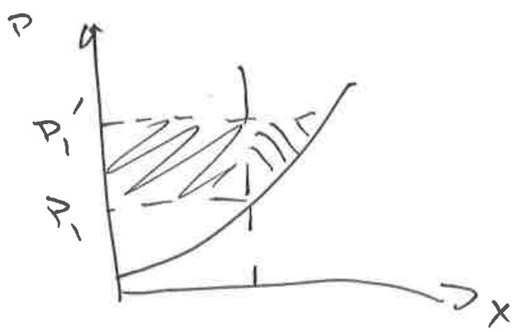
$$\Rightarrow EV = \underbrace{m - m'}_{} = 10 - 8.409 = \boxed{1.591}$$

EV is how much take away from original

Producer Surplus

→ Producer Surplus is area above supply curve and below price. This represents space above price willing to sell at and the price received.

Changes in PS



Benefit - cost analysis

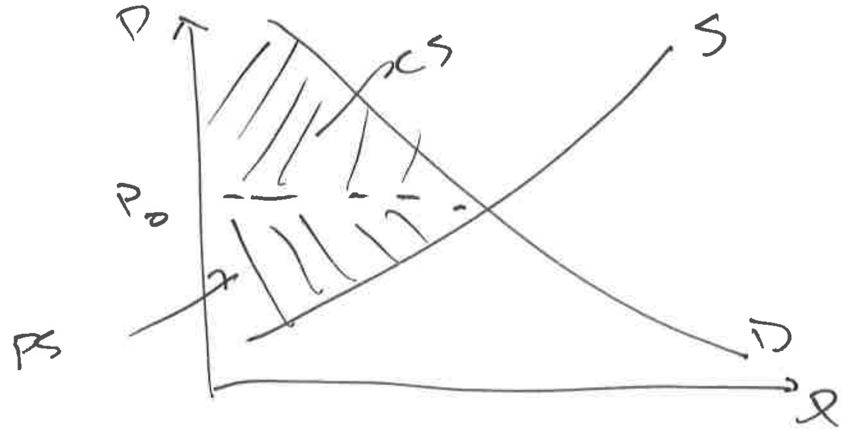
→ The tools we've learned in this chapter are useful for doing cost-benefit analysis

→ here we want to see impacts of changes in prices (e.g. b/c of taxes) on consumers' well-being

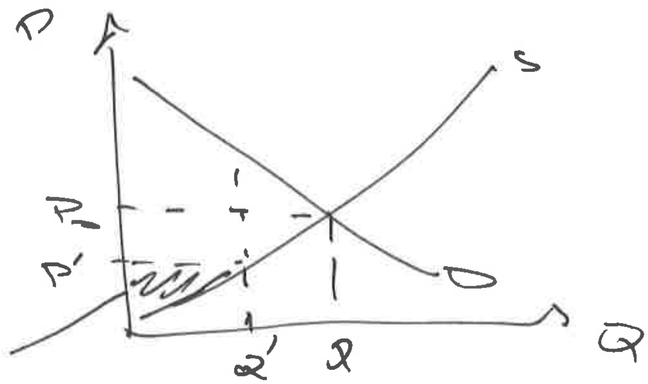
→ we can measure well-being by the CS, PS, EV, CV measures we've seen.

→ However, these measures don't always let us measure exactly the change in welfare, but they might at least allow us to put some bounds on what a reasonable change in welfare would be.

e.g. consider the market for medical care:



→ Suppose Bernie Sanders is able to win office and is able to pass ~~[illegible]~~ a single payer health care plan. Now prices of services are lower than the free mkt eq'm b/c the gov't uses its monopsony power.



Clearly, PS ↓

What about CS?

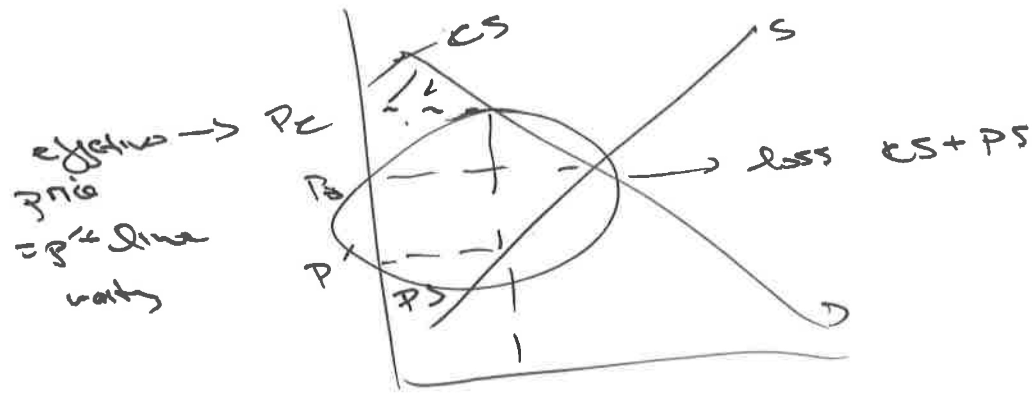
→ It depends on 2 things

1) who gets care (since quantity of care ↓, not everyone who got before gets now

2) what do people really "pay" to get care that suffers from a shortage?

→ e.g. costs to waiting in line for care?

If costs to lines:



→ and this assumes highest WTP demanders get care → and those left out have lowest WTP

If no cost to lines and care allocated to those w/ highest WTP: